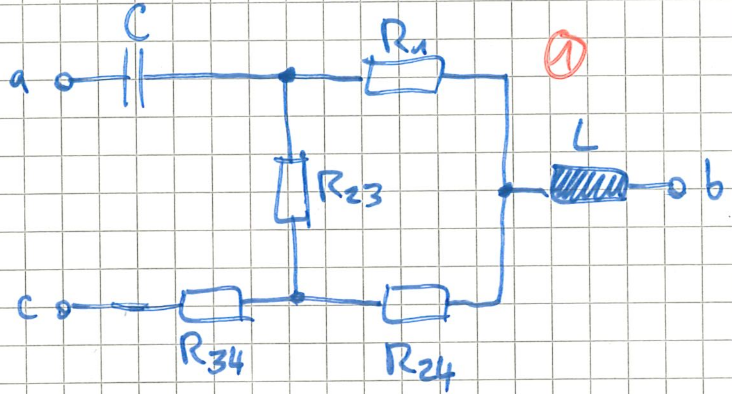
SS 2021

1a) $$\underline{Z}_{ab} = \frac{1}{j\omega C} + R_1 \| R_2 \| (R_3+R_4) + j\omega L$$ 1

$$= 0{,}5\,R \,\|\, (b_1+1)R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= \frac{\frac{b_1+1}{2}}{(b_1+1{,}5)} R + j\left(\omega L - \frac{1}{\omega C}\right)$$ 1

b)



$$R_{23} = \frac{R_2 \cdot R_3}{R_2+R_3+R_4} = \frac{1}{b_1+2} R$$

$$R_{34} = \frac{R_3 \cdot R_4}{R_2+R_3+R_4} = \frac{b_1}{b_1+2} R$$

$$R_{24} = \frac{R_2 \cdot R_4}{R_2+R_3+R_4} = \frac{b_1}{b_1+2} R$$ 1

$$\underline{Z}_{ac} = \frac{1}{j\omega C} + R_{23} \| (R_1+R_{24}) + R_{34}$$ 1

$$= \frac{R_{23}(R_1+R_{24})}{R_{23}+R_1+R_{24}} + R_{34} - j\frac{1}{\omega C}$$

$$= \frac{\frac{1}{b_1+2}\left(1+\frac{b_1}{b_1+2}\right)}{\frac{b_1+1}{b_1+2}+1} R + \frac{b_1}{b_1+2} R - j\frac{1}{\omega C}$$

$$= \left(\frac{2b_1+2}{(b_1+2)(2b_1+3)} + \frac{b_1}{b_1+2}\right) R - j\frac{1}{\omega C}$$ 1

c) $$\underline{Z}_{bc} = j\omega L + R_{24} \| (R_1+R_{23}) + R_{34}$$ 1

$$= \left(\frac{2b_1+2}{(b_1+2)(2b_1+3)} + \frac{b_1}{b_1+2}\right) R + j\omega L$$ 1

e) $$\omega L = \frac{1}{\omega C} \Rightarrow L = \frac{1}{\omega^2 C} = \mu \frac{w^2 h}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$$ 1

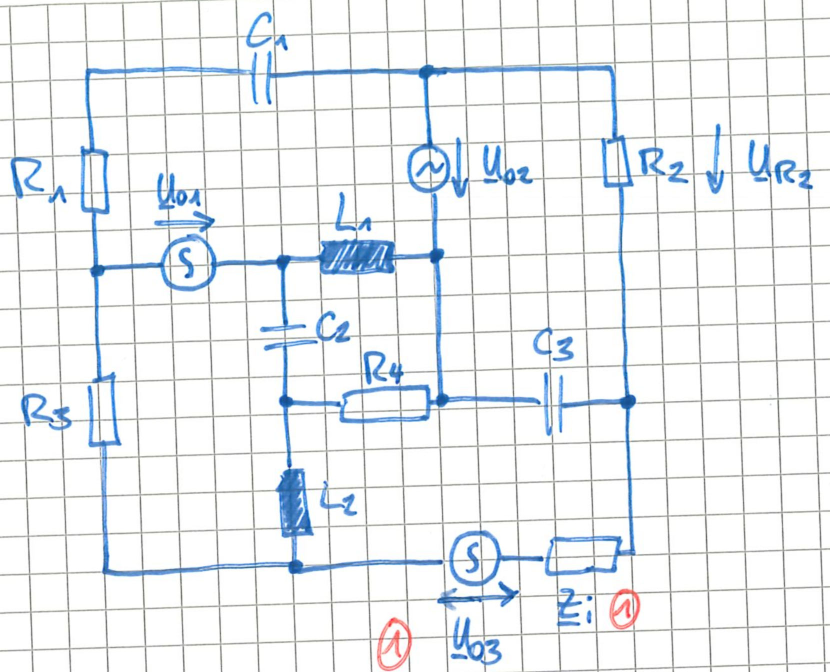
$$\Rightarrow w = \sqrt{\frac{1}{\omega^2 C} \cdot \frac{2\pi}{\mu h \ln\left(\frac{r_2}{r_1}\right)}} = 100{,}19$$

$$l = w \cdot \frac{1}{\frac{w}{l}} = 100{,}19 \cdot \frac{1}{10\frac{1}{cm}} = 10{,}02\text{ cm}$$ 1

d) $$L = \mu \frac{w^2 h}{2\pi} \ln\left(\frac{r_2}{r_1}\right) = \mu \frac{\left(l \cdot \frac{w}{l}\right)^2 h}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$$

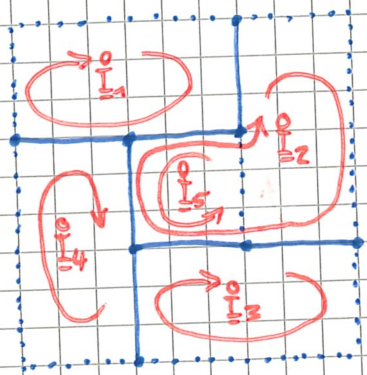
$$= 1\,\mu H$$ 1

2a)



$= \underline{I}_{03} \cdot \underline{Z}_i$

b)



... Graph ①

— Baum ①

c)

$m = z - k + 1 = 12 - 8 + 1 = 5$ ①

Maschendefinitionen ①

d)

$$\begin{pmatrix} R_1 + j(\omega L_1 - \frac{1}{\omega C_1}) & 0 & 0 & 0 & j\omega L_1 \\ -j\omega L_1 & R_2 + R_4 + j(\omega L_1 - \frac{1}{\omega C_2} - \frac{1}{\omega C_3}) & j\frac{1}{\omega C_3} + R_4 & j\frac{1}{\omega C_2} & -R_4 - j(\omega L_1 - \frac{1}{\omega C} \\ 0 & j\frac{1}{\omega C_3} & R_4 + j(\omega L_2 - \frac{1}{\omega C_3}) + \underline{Z}_i & -j\omega L_2 & R_4 \\ 0 & 0 & -j\omega L_2 & R_3 + j(\omega L_2 - \frac{1}{\omega C_2}) & -j\frac{1}{\omega C_{2}} \\ j\omega L_1 & 0 & R_4 & -j\frac{1}{\omega C_2} & R_4 + j(\omega L_1 - \omega \end{pmatrix}$$

$$= \begin{pmatrix} \underline{U}_{01} - \underline{U}_{02} \\ \underline{U}_{02} \\ \underline{I}_{03} \cdot \underline{Z}_i \\ -\underline{U}_{01} \\ 0 \end{pmatrix}$$

Diagonale ②
Nebenelemente ②
Spannungen ①

e) $\underline{I}_{R3} = \mathring{\underline{I}}_4$ ①

$\underline{U}_{R2} = R_2 \mathring{\underline{I}}_2$ ①

f) $R\,\underline{\mathring{I}}_1 + j10R\,\underline{\mathring{I}}_2 = -\underline{U}_{01}$

$j10R\,\underline{\mathring{I}}_1 + j10R\,\underline{\mathring{I}}_3 = 0 \Rightarrow \underline{\mathring{I}}_1 = -\underline{\mathring{I}}_3$

$j10R\,\underline{\mathring{I}}_2 + (b_2 + j10)R\,\underline{\mathring{I}}_3 = \underline{I}_{02}R$

$-jR\,\underline{\mathring{I}}_3 + j10R\,\underline{\mathring{I}}_2 = -\underline{U}_{01} \Rightarrow \underline{\mathring{I}}_2 = \dfrac{jR\underline{\mathring{I}}_3 - \underline{U}_{01}}{j10R}$

$\cancel{j10R}\,\dfrac{jR\underline{\mathring{I}}_3 - \underline{U}_{01}}{\cancel{j10R}} + (b_2 + j10)R\,\underline{\mathring{I}}_3 = \underline{I}_{02}R$

$\underline{\mathring{I}}_3 = \dfrac{\underline{I}_{02}R + \underline{U}_{01}}{(b_2 + j11)R} = \dfrac{1}{b_2 + j11} \cdot 36\,\text{mA}$ (1)

$\underline{\mathring{I}}_1 = -\dfrac{1}{b_2 + j11} \cdot 36\,\text{mA}$ (1)

$\underline{\mathring{I}}_2 = \dfrac{\underline{\mathring{I}}_3}{10} - \dfrac{\underline{U}_{01}}{j10R} = \dfrac{1}{b_2 + j11} \cdot 3{,}6\,\text{mA} + j\,2{,}4\,\text{mA}$

$= \left(\dfrac{3}{b_2 + j11} + j2\right) \cdot 1{,}2\,\text{mA}$ (1)

3a) $\underline{H}(j\omega) = \frac{\underline{U}_2}{\underline{U}_1} = \frac{j\omega L}{j(\omega L - \frac{1}{\omega C})} = \frac{\omega L}{\omega L - \frac{1}{\omega C}}$ 1

b) $|\underline{H}(j\omega)| = \frac{\omega L}{\omega L - \frac{1}{\omega C}} = \frac{\omega^2 L}{\omega^2 L - \frac{1}{C}}$ 1

$\varphi(\omega) = 0$ 1

c) $\omega = 0$:

$|\underline{H}(j\omega = 0)| = 0$ 1

$\varphi(\omega = 0) = 0$ 1

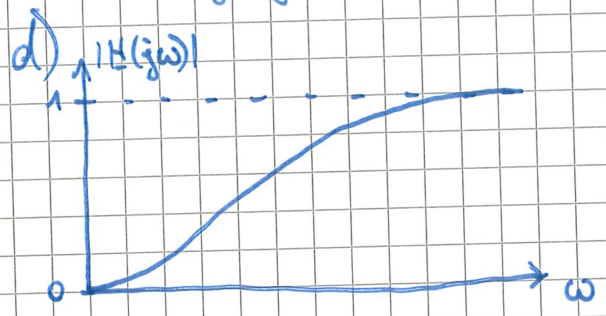
$\omega \to \infty$:

$|\underline{H}(j\omega \to \infty)| = 1$ 1

$\varphi(\omega \to \infty) = 0$ 1

Übertragungsverhalten: Hochpass 1

d)



1

1

e) $\underline{Z}_i = b_1 R + j\omega L \,||\, \frac{1}{j\omega C}$ 1 $= b_1 R + \frac{\frac{L}{C}}{j(\omega L - \frac{1}{\omega C})} = b_1 R - j \frac{L/C}{\omega L - \frac{1}{\omega C}}$ 1

f) $\underline{U}_L = \underline{U}_{01} \cdot \underline{H}(j\omega) = \underline{U}_{01} \cdot \frac{\omega L}{\omega L - \frac{1}{\omega C}}$ 1

$\underline{I}_k = \frac{\underline{U}_L}{\underline{Z}_i} = \frac{\underline{U}_{01} \frac{\omega L}{\omega L - \frac{1}{\omega C}}}{b_1 R - j \frac{L/C}{(\omega L - \frac{1}{\omega C})}} = \frac{\underline{U}_{01}\, \omega L}{b_1 R(\omega L - \frac{1}{\omega C}) - j\, L/C}$ 1

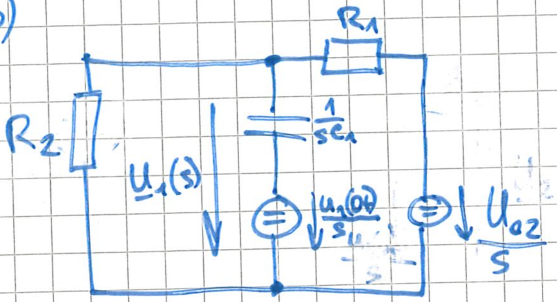
g) $\underline{U}_{Last} = \underline{U}_L \cdot \frac{\underline{Z}_{Last}}{\underline{Z}_{Last} + \underline{Z}_i}$ 1 $= \underline{U}_L \cdot \frac{\underline{Z}_i^*}{\underline{Z}_i^* + \underline{Z}_i}$

$= \underline{U}_{01} \frac{\omega L}{\omega L - \frac{1}{\omega C}} \cdot \frac{b_1 R + j \frac{L/C}{\omega L - \frac{1}{\omega C}}}{2 b_1 R}$

$= \frac{1}{2} \underline{U}_{01} \frac{\omega L}{\omega L - \frac{1}{\omega C}} \cdot \left(1 + j \frac{L}{C b_1 R(\omega L - \frac{1}{\omega C})}\right)$ 1

4a) $u_1(0+) = U_{01}$ 1

b)



2

c) $\underline{U}_1(s) = \underline{U}_{11}(s) + \underline{U}_{12}(s)$ 1

- $\underline{U}_{11}(s)$ bei $\frac{U_{02}}{s} = 0$:

$$\frac{\underline{U}_{11}(s)}{\frac{u_1(0+)}{s}} = \frac{R_1 \| R_2}{\frac{1}{sC_1} + R_1 \| R_2} \;(1)\; = \frac{\frac{R_1R_2}{R_1+R_2}}{\frac{1}{sC_1} + \frac{R_1R_2}{R_1+R_2}} = \frac{1}{\frac{1}{sC_1}\cdot\frac{R_1+R_2}{R_1R_2} + 1}$$

$$= \frac{s}{\frac{R_1+R_2}{C_1R_1R_2} + s}$$

$$\underline{U}_{11}(s) = \frac{u_1(0+)}{\cancel{s}} \cdot \frac{\cancel{s}}{s + \frac{R_1+R_2}{C_1R_1R_2}} = u_1(0+)\,\frac{1}{s+c} \quad \text{mit } c = \frac{R_1+R_2}{C_1R_1R_2} \;(1)$$

- $\underline{U}_{12}(s)$ bei $\frac{u_1(0+)}{s} = 0$:

$$\frac{\underline{U}_{12}(s)}{\frac{U_{02}}{s}} = \frac{R_2 \| \frac{1}{sC_1}}{R_1 + R_2 \| \frac{1}{sC_1}} \;(1)\; = \frac{\frac{\frac{R_2}{sC_1}}{R_2 + \frac{1}{sC_1}}}{R_1 + \frac{\frac{R_2}{sC_1}}{R_2 + \frac{1}{sC_1}}} = \frac{\frac{R_2}{sC_1}}{R_1\left(R_2 + \frac{1}{sC_1}\right) + \frac{R_2}{sC_1}}$$

$$= \frac{R_2}{R_2 + R_1(sC_1R_2 + 1)} = \frac{R_2}{sC_1R_1R_2 + R_1 + R_2}$$

$$= \frac{R_2}{R_1+R_2} \cdot \frac{R_1+R_2}{sC_1R_1R_2 + R_1 + R_2} = \frac{R_2}{R_1+R_2}\,\frac{\frac{R_1+R_2}{C_1R_1R_2}}{s + \frac{R_1+R_2}{C_1R_1R_2}}$$

$$\underline{U}_{12}(s) = U_{02} \cdot \frac{R_2}{R_1+R_2} \cdot \frac{c}{s(s+c)} \quad \text{mit } c = \frac{R_1+R_2}{C_1R_1R_2} \;(1)$$

d) $$u_1(t) = U_{01} \cdot e^{-\frac{R_1+R_2}{C_1R_1R_2}t} + U_{02}\,\frac{R_2}{R_1+R_2}\left(1 - e^{-\frac{R_1+R_2}{C_1R_1R_2}t}\right);\; t \geq 0$$

$$= U_{02}\,\frac{R_2}{R_1+R_2} + \left(U_{01} - U_{02}\,\frac{R_2}{R_1+R_2}\right) \cdot e^{-\frac{R_1+R_2}{C_1R_1R_2}t} \;;\; t \geq 0 \quad (1)$$

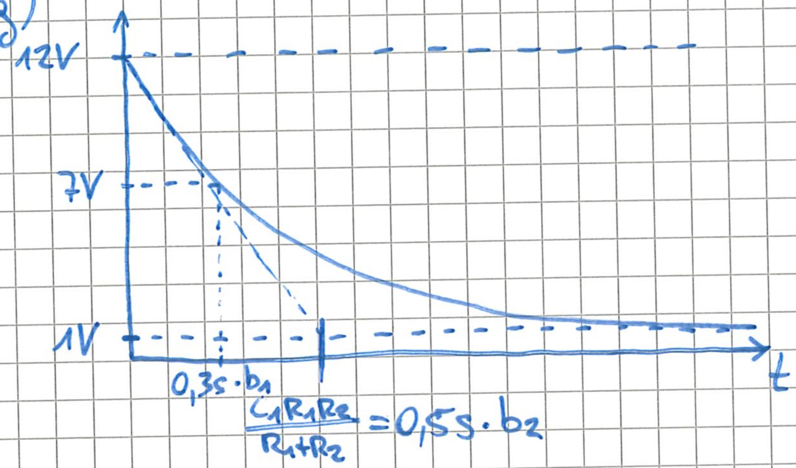
e) $u_1(t \to \infty) = U_{02}\,\frac{R_2}{R_1+R_2}$ 1 f)

$$u_1(t^*) = U_{02}\frac{R_2}{R_1+R_2} + \left(U_{01} - U_{02}\frac{R_2}{R_1+R_2}\right)\cdot e^{-ct^*} \quad \text{mit } c = \frac{R_1+R_2}{C_1R_1R_2}$$

$$\Rightarrow t^* = -\frac{C_1R_1R_2}{R_1+R_2}\ln\left(\frac{u_1(t^*) - U_{02}\frac{R_2}{R_1+R_2}}{U_{01} - U_{02}\frac{R_2}{R_1+R_2}}\right) \quad 1$$

$$= b_2 \cdot 0{,}3\,s \quad 1$$

g)



2